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Mechanical Coupling Effects on Turbomachine Mistuned Response

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Mistuning in turbine engine bladed disks often leads to mode localization, which can result in high vibratory stresses in a single group of blades. These stresses can lower the fatigue life of the blades. Therefore, understanding mistuning is essential for design of durable rotating machinery. This investigation provides one of the first experimental demonstrations of phenomena associated with mistuning, including frequency splitting and orthogonality. Results illustrate the role of internal coupling on mistuned response. Coupling appears to be dependent on fundamental mode shape. Strong coupling prevents localization in bending modes. However, in spite of weak internal coupling, localization does not occur in an observed torsion mode. The primary consequence of mistuning is a breakdown of orthogonality between the nodal patterns of the mode shapes and harmonic excitations, resulting in numerous resonant responses to a single harmonic forcing function. The breakdown of nodal patterns also leads to increased difficulty with mode description.

Nomenclature

$\boldsymbol{A}$ = undamped system matrix
F = magnification factor
f = forcing function
h = transverse geometry, i.e., thickness
$\boldsymbol{I}$ = identity matrix
j = $\sqrt{-1}$
k = stiffness
m = blade mass
N = number of blades
N_k = normalization of F_k
n = nodal diameter
p = engine order of excitation
$\boldsymbol{Q}$ = eigenvector or mode shape
q = blade displacement
$\boldsymbol{q}$ = displacement vector
R = dimensionless coupling strength
r = disk radial coordinate
t = time
Δf_i = mistuning of ith blade, $\Delta f_i = (\omega_{bi}^2 - \omega_b^2)/\omega_b^2$
ζ = viscous damping
θ = disk annular coordinate
λ = dimensionless eigenvalue
ν = modal participation factor
ρ = mass density of material
σ = interblade phase angle
ω = frequency

Subscripts

b = blade reference
c = coupling reference
i = blade index
k = mode index

Superscripts

c = cosine wave
s = sine wave
T = transpose
$*$ = complex conjugate

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Introduction

TURBINE engine bladed disks feature periodicity in the form of cyclic symmetry. As a result, these structures exhibit some remarkable dynamic properties. One of the most outstanding is the existence of extended modes. An extended mode means that the normal modes of vibration of a periodic structure take the form of a standing wave extending throughout the structure.

Periodic structures have been shown to be sensitive to disorder or imperfections that break down the periodicity. When this occurs, certain modes become localized in only a small portion of the structure. For bladed disks, the localized modes are limited to a single blade or small group of blades within the assembly. The consequence of this localization is a large degree of scatter in the vibration amplitudes of blades in compressor and turbine rotors. Moreover, individual blades often respond with much higher amplitude than predicted through classical tuned analyses. As a result, localization can seriously reduce the high-cycle fatigue life of these blades.

The imperfections or disorder that cause localization are often the result of uncontrollable circumstances, such as manufacturing tolerances. Although these variations in the blades have little effect on rotor performance, they can have a significant impact on the dynamic response of a rotor. This has become known as the classical mistuning problem because these imperfections are evident as small variations in the natural frequencies of the individual blades.

Several issues associated with mistuning are investigated in this paper. First, the effect of mistuning on the eigenvalue problem of the tuned system is examined. Using this as a basis, the role of orthogonality on the mistuned response is discussed. Bench tests, as well as rotating tests, of a compressor fan rotor are used to substantiate mathematical analyses, yielding one of the first detailed experimental demonstrations of rotor mistuning phenomena. Results from these experiments are used to evaluate mechanical coupling in terms of internal coupling between substructures. The interaction of bending and torsion modes is also considered.

Formulation of Mistuning Problem

The most important features of mistuning and mode localization can be demonstrated using a simple model, such as the nearly periodic structure shown in Fig. 1. This model consists of a cyclic assembly of single degree-of-freedom (DOF) oscillators, and has

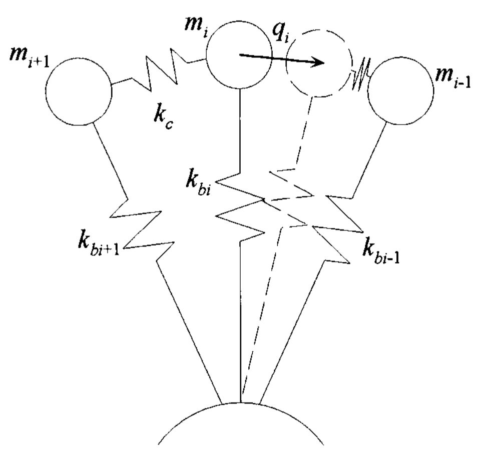

Fig. 1 Geometry of simple, nearly periodic structure.

been used repeatedly in the literature to demonstrate the effects of mistuning.[1,2] For a bladed disk, each oscillator represents a blade with a mass m_i and stiffness k_{bi}. The single DOF motion is a single-mode approximation of one of the blade natural modes, i.e., bending or torsion. The oscillators are coupled through an additional spring k_c, which can represent the effect of hub stiffness, a midspan shroud, or even motion-dependent aerodynamic loads.

Following the derivation of Wei and Pierre,[2] the governing equations of free, undamped motion can be shown to be

$$\ddot{q}_i + \omega_{bi}^2 q_i + 2\omega_c^2 q_i - \omega_c^2 q_{i-1} - \omega_c^2 q_{i+1} = 0, \qquad i = 1, \ldots, N \tag{1}$$

To specify cyclic symmetry, $q_0 = q_N$ and $q_1 = q_{N+1}$. Defining

$$\omega_{bi}^2 = \omega_b^2(1 + \Delta f_i), \qquad i = 1, \ldots, N \tag{2}$$

$$R^2 = \omega_c^2 \big/ \omega_b^2 \tag{3}$$

Eq. (1) can be written in matrix form,

$$\ddot{\boldsymbol{q}} + \omega_b^2 \boldsymbol{A}\boldsymbol{q} = \boldsymbol{0} \tag{4}$$

where

$$\boldsymbol{A} = \begin{bmatrix} A_1 & -R^2 & 0 & \cdots & -R^2 \\ -R^2 & A_2 & -R^2 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -R^2 & 0 & \cdots & -R^2 & A_N \end{bmatrix} \tag{5}$$

and $A_i = 1 + 2R^2 + \Delta f_i (i = 1, \ldots, N)$. Assuming simple harmonic motion, $\boldsymbol{q} = \boldsymbol{Q}e^{j\omega t}$, the modes of free vibration can be obtained from the eigenvalue problem

$$(\boldsymbol{A} - \lambda \boldsymbol{I})\boldsymbol{Q} = 0 \tag{6}$$

Localization in mistuned response comes from the properties of $\boldsymbol{A}$. If $\Delta f_i = 0$, i.e., the system is tuned and all blades have the same natural frequency ω_b, then $\boldsymbol{A}$ is circulant; i.e., each row is a circular shift right by one position of the previous row. Formulation of the eigenvalues and eigenvectors is given in Bendikson[1] and in Wei and Pierre.[2] The orthonormal eigenvectors are restated here for N even by

$$\left\{\boldsymbol{Q}_k^c\right\}^T = \{1, \cos\sigma_k, \ldots, \cos(N-1)\sigma_k\}, \qquad k = 1, \ldots, N/2 + 1 \tag{7}$$

$$\left\{\boldsymbol{Q}_k^s\right\}^T = \{0, \sin\sigma_k, \ldots, \sin(N-1)\sigma_k\}, \qquad k = 2, \ldots, N/2 \tag{8}$$

where $\sigma_k = 2\pi(k-1)/N$. The corresponding eigenvalues are given by

$$\lambda_k = \omega_k^2 \big/ \omega_b^2 = 1 + 2R^2\{1 - \cos[2\pi(k-1)/N]\} \qquad k = 1, \ldots, N/2 + 1 \tag{9}$$

The integer k is physically related to the number of nodal diameters n occurring in the response by $n = k - 1$. Except for $k = 1$ and $k = N/2 + 1$, the eigenvalues are repeated, i.e., there are two eigenvalues for each k, one corresponding to $\boldsymbol{Q}_k^c$ and the other corresponding to $\boldsymbol{Q}_k^s$. The actual response in a bladed disk can be any linear combination of both eigenvectors occurring at the same frequency. For N odd, $N/2 + 1$ in Eqs. (7–9) is replaced by $(N + 1)/2$, and there is one simple eigenvalue for $k = 1$.

The repeated eigenvalues and corresponding sine and cosine modes are a property of circulant matrices. It is important to note that the mode shapes for this type of system are extended, such that the wave motion of the response extends throughout the entire structure. Mistuning, i.e., $\Delta f_i \neq 0$, disrupts the circulant nature of the matrix. Assuming that each Δf_i is distinct, repeated eigenvalues no longer occur. Instead, there are N simple eigenvalues, each corresponding to a distinct mode shape.

The nature of each mode shape in terms of the tuned mode shapes depends on the coupling strength R, and has been discussed in detail by Wei and Pierre.[2] To summarize their results, for small R, the mode shapes change drastically when small mistuning is introduced. Response in this case becomes strongly localized because the disruption of the extended mode shape prohibits the wavelike propagation of response throughout the structure.[3] However, for strongly coupled systems with small mistuning, large R, the frequencies of the individual modes tend to be very close to the frequency pairs of the tuned system, and the mode shapes are perturbations of the tuned system mode shapes. As a result, two responses with a given n are likely to occur at slightly different frequencies. This phenomenon is known as frequency splitting.

This analysis has been applied to a simple system with one DOF per sector. However, the same analysis may be applied to a multiple DOF system without significantly altering the result. In this case, $\boldsymbol{A}$ becomes block-circulant, and the solution of the eigenvalue problem yields families of modes, each family similar to the modes in the single DOF system. Treatment of the multi-DOF case is given in Bendikson.[1]

Mode Participation and Orthogonality

A bladed disk behaves much like a circular plate in terms of its system mode shapes. Therefore, the mathematics used to describe vibrations in a circular plate may be applied to examine orthogonality. To do this, an annular coordinate θ and a radial coordinate r are adopted with origin at the disk axial centerline. Axial, or through-the-thickness, effects are assumed to be negligible to cast the problem in two dimensions. Finally, only out-of-plane deflections are considered.

When a structure is excited harmonically, it responds with motion that is a linear combination of all of its natural modes. Defining the displacement at a given mode k as

$$\boldsymbol{q}_k(r, \theta, t) = \boldsymbol{Q}_k(r, \theta)e^{j\omega_k t} \tag{10}$$

each mode participates with different intensity ν_k, resulting in an overall motion of

$$\boldsymbol{q}(r, \theta, t) = \nu_1 \boldsymbol{q}_1(r, \theta, t) + \nu_2 \boldsymbol{q}_2(r, \theta, t) + \cdots \tag{11}$$

Thus it can be seen that the intensity of the response at mode k that participates in the motion of the structure depends on ν_k, the modal participation factor. This term is defined as

$$\nu_k = F_k \Bigg/ \omega_k^2 \sqrt{\left[1 - (\omega/\omega_k)^2\right]^2 + 4\zeta_k^2(\omega/\omega_k)^2} \tag{12}$$

For well-spaced modes, the denominator of this expression insures that modes well away from a given frequency of interest play little part in the response. For closely spaced modes, however, the numerator becomes the most important term in this expression. This term F_k is dependent on mode shape, excitation force location, and distribution.[4]

$$F_k = \frac{1}{\rho h N_k} \int_\theta \int_r f_p^* \boldsymbol{Q}_k \, \mathrm{d}r \, \mathrm{d}\theta \tag{13}$$

$$N_k = \int_\theta \int_r \boldsymbol{Q}_k^2 \, \mathrm{d}r \, \mathrm{d}\theta \tag{14}$$

For periodic, i.e., circular, structures, closely spaced modes may be described by the number of nodal diameters n that appear in the response. Such mode shapes are generally independent of r. Recalling that $n = k - 1$, the modal response may be written in terms of nodal diameter, $\boldsymbol{Q}_{k-1} = \boldsymbol{Q}_n(\theta)(k = 1, \ldots, N/2 + 1)$. In this case, the primary forcing functions of interest and those being studied here are functions of θ. Therefore, it may be assumed for this study that $f = f(\theta)$. Based on these simplifications, Eq. (13) may be rewritten:

$$F_n = \frac{1}{\rho h N_n} \int_\theta f_p^*(\theta) \boldsymbol{Q}_n(\theta) \, \mathrm{d}\theta \tag{15}$$

In a cyclically symmetric structure such as a tuned bladed disk, n is an integer, and $\boldsymbol{Q}_n(\theta)$ may be assumed to be sinusoidal:

$$\boldsymbol{Q}_n(\theta) = \boldsymbol{Q} e^{jn\theta} \tag{16}$$

Furthermore, certain "well-behaved" forcing functions such as those considered in this study may be described by sinusoidal functions in terms of the integer number of engine orders p of the excitation:

$$f_p(\theta) = f e^{jp\theta} \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (15) and integrating over the rotor annulus yields

$$F_n = \frac{f\boldsymbol{Q}}{\rho h N_n} \int_0^{2\pi} e^{-jp\theta} e^{jn\theta} \, \mathrm{d}\theta \begin{cases} = 0, & p \neq n \\ \neq 0, & p = n \end{cases} \tag{18}$$

This result indicates that integral engine-order excitations and disk modes with sinusoidal annular nodal patterns are orthogonal. This orthogonality dictates that modes described by nodal diameter n are excited by forcing functions of engine-order p only if $p = n$ for well-behaved systems with cyclic symmetry. However, if the nodal diameters of the closely spaced modes become skewed such that cyclic symmetry is no longer a valid assumption and n is no longer an integer, then the mode shape at mode k can no longer be described using a sinusoid

$$\boldsymbol{Q}_n(\theta) \neq \boldsymbol{Q} e^{jn\theta} \tag{19}$$

As a result, orthogonality between f and $\boldsymbol{Q}_n$ breaks down, and a p engine-order excitation can excite an entire family of closely spaced modes, resulting in numerous resonant responses in a small speed and frequency range for a bladed disk.

Description of Experiment

The bladed disk used in this investigation is the first stage of a two-stage transonic compressor. It is a Ti-6-4 integrally bladed disk, or blisk, consisting of 16 low aspect ratio airfoils on an integral stub shaft. The diameter of the blisk is approximately 0.7 m. Its aerodynamic performance and basic mechanical properties are well documented.[5] A Campbell diagram showing the predicted response of the blisk is given in Fig. 2. The primary mode families of interest in this study are an isolated first bending (1B) family, and closely spaced second bending (2B) and first torsion (1T) families of modes.

The blisk was tested in a static bench condition and under operating conditions. For bench testing, the blisk was mounted using a balance arbor in the hub bore to produce an evenly applied boundary condition. A piezoelectric crystal was mounted on the test stand and used to excite the blisk at resonance. Digital holography captured

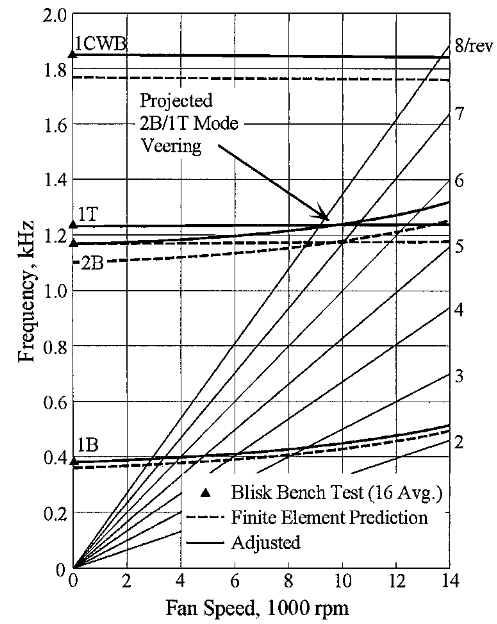

Fig. 2 Blisk Campbell diagram.

the mode shapes of the blisk in the three mode families of interest. The frequency resolution of the excitation and holography system at the 1B family of modes was 0.1 Hz, and at the 2B and 1T families the frequency resolution was 1 Hz.

In addition, a simplified modal analysis of the blisk was performed using accelerometers mounted on the blade tips at the leading and trailing edges to differentiate between bending and torsion. The blisk was excited using a magnetic exciter attached to one of the blade tips. Data were recorded and analyzed using a standard modal analysis software package.

The blisk was also tested under operating conditions. The 1B family of modes was excited using a $p = 3$ engine-order excitation, and the 2B and 1T mode families were excited with a $p = 8$ excitation. Excitations were provided by total pressure distortions introduced in the compressor inlet flow. Similar tests have been described in detail, including the excitation method, in previous studies.[6,7] It should be noted that the $p = 8$ forcing function excites the blisk near a 2B/1T eigenvalue veering region, which has been studied by Kenyon.[8] Response of the blades during operational testing was obtained from dynamic strain gauges fixed at the root of each blade. Analog signals were recorded on tape as the rotor was accelerated through the resonant conditions, and then sampled at 20 kHz with a 3-kHz low-pass filter. The power spectra and cross spectra of the transient signals were obtained using a contiguous block averaging method with a frequency resolution of 0.5 Hz. Interblade phase angles were computed from the cross spectra of adjacent blades.

Frequency Splitting at 1B Mode

Frequency splitting was observed at the 1B family of modes using holographic techniques and modal analysis. In this mode family, regularly spaced node lines were evident for all possible nodal diameters, $n = 0, \ldots, 8$ for a 16-bladed rotor. The frequencies at which each mode shape occurred are shown in terms of nodal diameter in Table 1. The results for $n = 2$–6 are in excellent agreement for the two test techniques. Data from $n < 2$ and $n > 6$ were not available from modal analysis.

The appearance of frequency pairs for $n = 3$–7 is indicative of the frequency splitting phenomenon, which has been shown mathematically to occur as a result of mistuning. The mode shapes for $n = 3$ are shown in Fig. 3. The two modes are similar in appearance except for a phase difference. The node lines do not quite appear evenly spaced, which is a common effect of mistuning, but an approximate $n = 3$ pattern is evident.

Testing of the blisk under operating conditions with a $p = 3$ forcing function produced resonant response consistent with bench

Table 1 1B frequencies

n	ω, Hz—Modal analysis	ω, Hz—Holography
0	——	361.0
1	——	376.2
2	378.2	378.1
3	380.6	380.0, 381.0
4	382.0	381.9, 382.2
5	382.6, 382.8	384.6
6	383.2	385.2, 385.6
7	——	414.0, 418.0
8	——	449.0

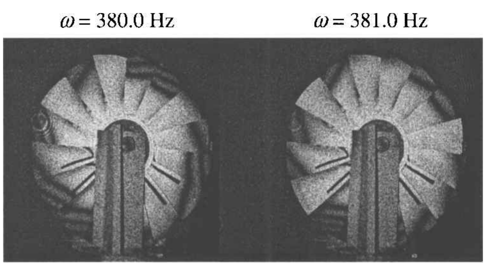

Fig. 3 $n = 3$ mode shapes.

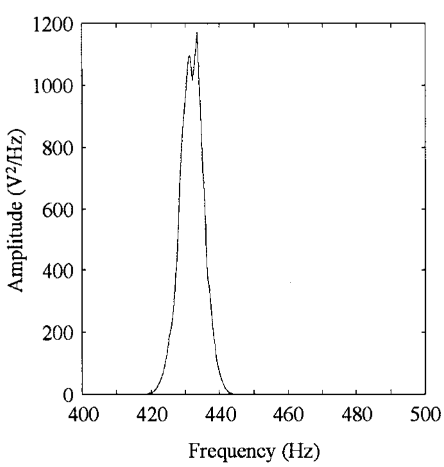

Fig. 4 Blisk response to 3/revolution forcing function.

test findings. The frequency spectrum from one of the blade strain gauges is shown in Fig. 4. Two distinct resonant peaks can be seen in the response at $\omega = 431.5$ and 433.5 Hz. Differences in these frequencies from bench test frequencies are the result of centrifugal loading at speed. Distinct mode pairs such as those shown in the figure were evident in the majority of the blades during the acceleration through the 1B mode family.

The mode shapes were examined using interblade phase angle σ_k, which was computed from the cross spectra of adjacent blades at each of the two modes. Recalling that $\sigma_k = 2\pi(k-1)/N = 2\pi n/N$ and converting to degrees, $\sigma = 67.5$ deg indicates an $n = 3$ mode shape. For the blade whose response is shown in Fig. 4, $\sigma = 55$ deg for the first mode and $\sigma = 49$ deg for the second mode. The change in σ is small between modes, indicating that the mode shapes are approximately the same for both modes. Moreover, other blades on the blisk showed similar response at the two modes. Ranging from approximately 50 to 75 deg, σ was either slightly less than or slightly more than the expected 67.5 deg for $n = 3$. The variation in σ above and below the expected value is consistent with bench test results indicating that nodal lines are not evenly spaced. This results in some phase variation in the response. It should be noted that only the $n = 3$ mode in the 1B family responded when the blisk was excited by a $p = 3$ excitation. The other modes in the 1B family did not respond, indicating that these were orthogonal to the forcing function.

The response of the blisk at the 1B family of modes is essentially a perturbation of the tuned system response. Based on the development of the mistuning problem and related discussion on R, this indicates that the blisk exhibits strong internal coupling, or high R. Such coupling prevents strong localization in a rotor, which reduces the consequences of mistuning. Significant stress variations can still occur in a blisk with high R, but these can generally be attributed to other factors, such as variations in unsteady aerodynamic damping, and are less severe.[7]

Coupling at 2B and 1T Modes

Internal coupling was further investigated at the 2B and 1T mode families in a bench condition using holographic and modal testing techniques. Natural frequencies for the 2B family of modes are shown in Table 2. These frequencies are referenced by a generic mode number rather than nodal diameter because nodal diameters were not always clearly identifiable. The $n = 0$ and 1 modes were identified as modes 1 and 2 in the table using holographic techniques at 1110 and 1121 Hz, respectively. For modes with higher frequencies, the node lines appear more irregular, such as shown in Fig. 5. For each of the modes in the figure, at least one nodal diameter is evident. However, several other node lines are evident, either as skewed diameters or as radii from the blisk center. It should also be noted that these modes are very closely spaced, only 1 Hz apart, though the mode shapes show distinct differences.

The modes in the 1T family were even more difficult to characterize. These are listed in Table 3. No modes with an integer n were observed. Instead, node lines in the blisk were irregular and skewed, as shown in Fig. 6. The modes shown in the figure are very

Table 2 2B frequencies

Mode	ω, Hz
1	1110
2	1120, 1121
3	1176, 1177
4	1183, 1184
5	1213, 1215

Table 3 1T frequencies

Mode	ω, Hz
1	1234
2	1236
3	1237
4	1238
5	1240

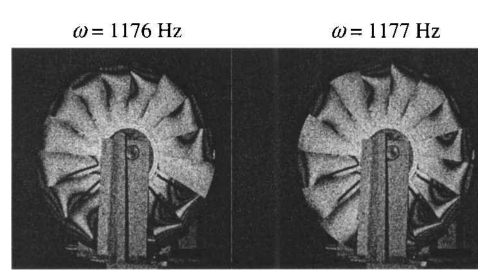

Fig. 5 Closely spaced 2B modes.

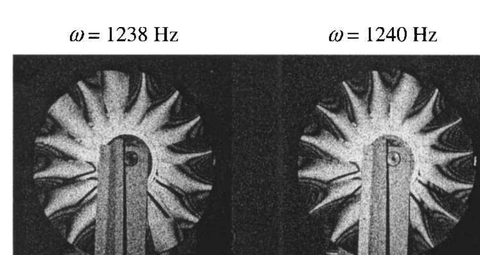

Fig. 6 Closely spaced 1T modes.

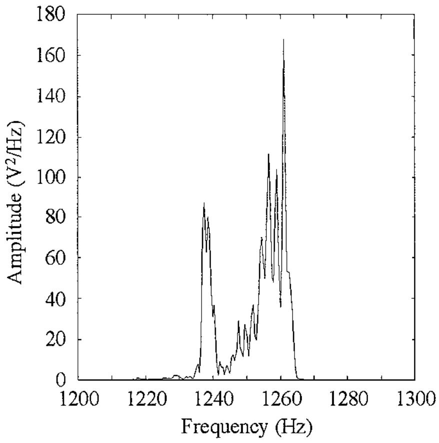

Fig. 7 Blisk 2B/1T response to 8/revolution excitation.

Table 4 σ_k for select blade pairs at 1T modes

Blade pair	ω = 1259 Hz	ω = 1261 Hz
Blades 4–5	−133.0 deg	−172.1 deg
Blades 5–6	145.5 deg	149.5 deg
Blades 6–7	140.1 deg	——
Blades 7–8	174.4 deg	——
Blades 12–13	−148.5 deg	−161.7 deg
Blades 13–14	−173.2 deg	166.1 deg

ω = 1224 Hz ω = 1225 Hz

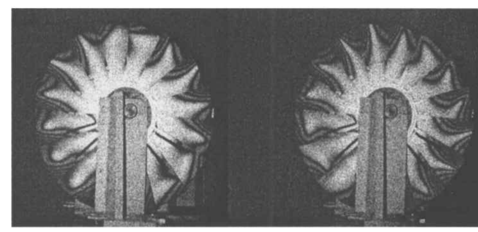

Fig. 8 Mixed 2B and 1T modes.

closely spaced, yet have entirely different mode shapes. As seen in Table 3, the 1T modes do not occur in discrete pairs as for the bending modes. A group of five modes with distinct shapes appeared in a small frequency range.

Based on these results, the coupling value R appears to remain relatively high for the 2B mode, as evidenced by the discrete mode pairs. However, the lack of similarity in the mode shapes at higher frequencies in the 2B family suggests that R may not be as high as for the 1B mode family. These results further indicate that R is quite low for the 1T modes because mode pairs do not occur and nodal patterns are skewed. Thus R is shown to be mode dependent. Physically this implies that a different mechanism couples the blades for each mode. The similarity in R of the two bending modes suggests that the mechanism that influences R may be based on the fundamental blade mode shape.

From the discussion on orthogonality, the mode shapes of a mistuned system with significant mode distortion may not be orthogonal to a harmonic forcing function. Such a loss of orthogonality to a harmonic excitation was demonstrated at the 2B and 1T modes under operating conditions. The blisk was excited at these modes using a $p = 8$ engine-order forcing function. With well-defined, evenly distributed mode shapes, this should result in a single $n = 8$ response for each of the two mode families for an $N = 16$ blisk. However, orthogonality to the harmonic forcing function was disrupted by mistuning, yielding numerous 2B and 1T resonant responses. The frequency spectra from one of the blade-mounted strain gauges is shown in Fig. 7 for an acceleration through the resonant conditions. The figure clearly shows numerous peaks corresponding to modes in the response. Assuming that the two primary clusters of modes are 2B modes at the lower frequencies and 1T modes at the higher frequencies, at least two or three bending modes and three or four torsion modes are evident. The two primary bending modes in the frequency spectrum are not distinct. They appear to be a frequency pair, similar to those at the 1B mode though the mode shapes are not necessarily similar as evidenced by bench test results. This implies that the response is not the simple $n = 8$ mode as expected for a tuned system, but rather a pair corresponding roughly to lower n from the tuned system that was excited because of the breakdown in orthogonality. The torsion modes are predominantly distinct, confirming that these do not occur in pairs as for the bending modes. In addition to the 2B and 1T clusters, several modes of smaller amplitude can be seen in the full range of frequencies between these clusters.

Mode shapes were again investigated using σ. Wide variations in σ confirmed that node lines were very irregular, and that the mode shapes changed significantly between closely spaced modes. This was evident for the 1T modes, particularly closely spaced modes occurring at $\omega = 1259$ and 1261 Hz. Values of σ available from particular blade pairs at these modes are shown in Table 4. Two important observations can be made. First, σ is inconsistent for a given mode from one blade pair to another. This indicates that the node lines are irregularly spaced at these modes. Second, σ changes dramatically from $\omega = 1259$ to 1261 Hz for most of the blade pairs shown, demonstrating that the mode shapes are dissimilar although the modes are closely spaced. These observations are consistent with bench test results and reaffirm that the blisk exhibits a low R value at the 1T modes, resulting in a loss of orthogonality to harmonic excitations.

It should be noted that localization was not observed at the 2B or 1T modes. Localization is generally recognized as the most significant consequence of mistuning. The blisk has been shown to exhibit the characteristics of a mistuned system, including frequency splitting for high R and strong mode distortion for low R, but does not evince localization. This may be attributed to the relatively high R value for the 2B modes. However, this absence of localization occurs in the 1T mode family in spite of the lack of strong coupling. Instead, the most significant effect of mistuning observed in the blisk is the appearance of multiple responses to a single harmonic forcing function because of the loss of orthogonality to the excitation source.

2B and 1T Mode Interaction

The response at the 2B and 1T mode families was further complicated by bending and torsion modal interaction. Such interaction has been described mathematically by Yang and Griffin,[9] and occurs when the natural frequencies of bending and torsion mode families are close to each other. In the blisk, it occurs as a result of an eigenvalue veering, and has been described in detail for the nominal blade geometry of the blisk by Kenyon.[8]

Significant interaction was seen in a mode pair at $\omega = 1224$ and 12254 Hz with the rotor in a bench condition. Holograms of these modes are shown in Fig. 8. Individual blades can been seen in both bending and torsion for each of these modes. Several blades also exhibit "mixed" mode shapes at both frequencies. An additional mode with bending/torsion interaction was also observed at $\omega = 1227$ Hz. The blade mode shapes for these modes are similar to those in Kenyon[8] for this blade geometry at various stages of the veering.

These results were verified using modal analysis. With accelerometers at the blade tip leading and trailing edges, bending and torsion could be discerned from phase. When the leading and trailing edges were in phase, the blade was moving in bending. When the leading and trailing edges were approximately 180 deg out of phase, the blade was in torsion. Blades with intermediate phase angles between the leading and trailing edges were generally in a mixed mode. Both of the modes shown in Fig. 8 were identified using this modal analysis technique.

The result of the interaction is increased difficulty in mode identification and measurement. Thus far σ has been used to examine mode shapes at the various mode families in terms of nodal patterns. However, mode interaction means that the measured σ includes terms from both the blisk nodal patterns, as well as phase differences from bending and torsion, as demonstrated through modal analysis. Therefore, it is unlikely that σ measured at intermediate modes, such as those between the strong 2B and 1T clusters in Fig. 7, will be sufficient for identifying these modes. Moreover, mode shapes are likely to change further as a function of speed with the eigenvalue veering.

Conclusions

The mathematical development of the mistuning problem was summarized with treatment given to frequency splitting and orthogonality. These phenomena were demonstrated in a compressor fan using static bench tests and rotating tests under normal operating conditions. Frequency splitting was shown at the 1B family of modes, and to a lesser extent, at the 2B family. Distortion of nodal patterns due to mistuning was demonstrated at both the 2B and 1T mode families. This distortion caused a breakdown in the orthogonality of these mode shapes to harmonic forcing functions as indicated through testing under operating conditions.

The effect of internal coupling on the mistuned response of the blisk was investigated. Both bending modes exhibited strong coupling, as evidenced by discrete frequency pairs. In the case of the 1B mode family, the individual modes in each frequency pair had similar mode shapes. For the 2B family, this was true of some of the mode pairs, but not for all, indicating that coupling is stronger for the 1B mode. The 1T mode family demonstrated weak coupling through strong nodal distortion and a lack of discrete frequency pairs. These results suggest that coupling is dependent on fundamental mode shape. Strong internal coupling prevented localization in the mistuned resonant response of both bending modes. Localization was also not observed in the 1T mode, though this cannot be attributed to coupling. The primary consequence of mistuning in the blisk was the breakdown of harmonic orthogonality, resulting in numerous resonant responses to a single harmonic excitation.

Interaction between the 2B and 1T mode families was shown. The primary consequence of this interaction was that interblade phase angle was no longer a valid means of identifying nodal patterns in the disk, thereby making mode identification more difficult.

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